On multiple zeta values and Bernoulli numbers

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Multiple zeta values $\zeta(\boldsymbol{k})$ and $\zeta^*(\boldsymbol{k})$, which are two of various natural generalizations of Riemann zeta values, are defined as follows. For any multiple index $\boldsymbol{k} = (k_1, k_2, \ldots, k_n)$ ($k_i \in \mathbf{Z}$, $k_i > 0$), the weight and height of $\boldsymbol{k}$ are by definition the integers $k = k_1 + k_2 + \cdots + k_n$ and $s = \#\{i|\ k_i > 1\}$, respectively. Index $\boldsymbol{k} = (k_1, k_2, \ldots, k_n)$ is said to be admissible if its first entry satisfies the extra requirement $k_1 \geq 2$. For each admissible multiple index $\boldsymbol{k}$, we define two kinds of multiple zeta values by

$$\zeta(\boldsymbol{k}) = \zeta(k_1, k_2, \ldots, k_n) = \sum_{m_1 > m_2 > \cdots > m_n > 0} \frac{1}{{m_1}^{k_1} {m_2}^{k_2} \cdots {m_n}^{k_n}}$$

and

$$\zeta^*(\boldsymbol{k}) = \zeta^*(k_1, k_2, \ldots, k_n) = \sum_{m_1 \geq m_2 \geq \cdots \geq m_n \geq 1} \frac{1}{{m_1}^{k_1} {m_2}^{k_2} \cdots {m_n}^{k_n}} .$$

Multiple zeta values normally mean $\zeta(\boldsymbol{k})$ in literatures, and Euler was interested in $\zeta^*(\boldsymbol{k})$. In this talk, three families of relations between sums of multiple zeta values ζ^* and Riemann zeta values are planning to be given. One of them is as follows.

Theorem (with Takashi Aoki) Let k and s be integers such that $k/2 \geq s \geq 1$. Let $I_0(k,s)$ denote the set of all admissible multiple indices of weight k and height s. Then we have

$$\sum_{\boldsymbol{k} \in I_0(k,s)} \zeta^*(\boldsymbol{k}) = 2\binom{k-1}{2s-1}(1-2^{1-k})\zeta(k).$$

Farthermore, a proof of certain kind of formula of Bernoulli numbers is given as a consequence of their theorems.