An attempt to interpret the Weil explicit formula from Beurling's spectral theory

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Let $\rho = \beta + i\gamma$ be the non-trivial zeros of the Riemann zeta-function.

In a previous study we considered the quantity

$$\sum_{\rho} e^{u\rho^2 - v\rho}, \quad u > 0, \quad v \in \mathbf{R}, \tag{1}$$

and obtained asymptotic formulas as $u \to 0+$. We saw that the quantity (1) behaves quite differently as $u \to 0+$, according as $v = 0$, $v = \mp \log p^m$ (p is a prime), or otherwise.

A motivation of considering the quantity (1) is as follows. To make the explanation simple, let us consider the quantity

$$\sum_{\rho} e^{u(\rho-1/2)^2 - v(\rho-1/2)}$$

instead of (1), and let us assume the Riemann hypothesis. Then this is equal to

$$\sum_{\gamma} e^{-u\gamma^2 - iv\gamma}. \tag{2}$$

A. Beurling made great contributions to the problem of *spectral synthesis*. Roughly speaking, it is the problem for approximating a function ϕ by trigonometric polynomials in some topology. He introduced the transform

$$U_\phi(u, v) = \int_{-\infty}^{\infty} \phi(t) e^{-u|t| - ivt} dt, \quad u > 0, \quad v \in \mathbf{R}. \tag{3}$$

According as the behavior for $U_\phi(u, v)$ as $u \to 0+$, the concept of *spectral sets* is introduced. Roughly speaking, the spectral set of ϕ is defined to be the complement of the set of v for which $U_\phi(u, v) \to 0$ as $u \to 0+$. Spectral sets play a fundamental role in the problem of spectral synthesis.

We felt some resemblance between (2) and (3). This was a motivation of considering (1).

In this talk we will discuss a more general quantity than (1) and show asymptotic results for it.

This is a joint work with Masatoshi Suzuki.