We will give a characterization of those modular functions on subgroups of finite index of the full modular group with divisors supported at the cusps in terms of the growth of the exponents in their q-product expansions.